Title

A theorem in triangles: Median drawn to the hypotenuse of a right triangle

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Short description/ main idea

In this scenario the pupils are called to understand the theorem through: a) its proof, (thus proving the theorem is one of the main goal states of the lesson), b) the effective usage of the geogebra applets (which are especially designed by the teacher for this purpose) and c) through guided discovery and exploration of real life problems.

Typical starting point of the exploratory learning approach is to engage students while introducing the problem by asking a challenging question or by presenting to them a problematic situation that touches upon their experiences and interests.

During this lesson, the students are expected to collaborate in order to a) prove the theorem, b) apply the theorem in order to solve mathematical problems and c) create their own mathematical problems (using their real-life experience and the newly-acquired knowledge) that can be solved with this theorem.

In this scenario, it is assumed that students are familiarised with the use of computers in general and the use of the Geogebra software (dynamic geometry tool), in particular. Thus, a typical entry behaviour is that students know how to handle the functionalities of the software.

Learning objectives

Cognitive - Process: To remember, To understand, To apply, To think critically and creatively

Learning activities

Phase 1: Diagnostic phase

Description of phase: There is a laptop above every desk and the students are working mostly in pairs, except for the proof of the theorem, where a student with the guidance and support of the teacher demonstrates to the whole class the proof of the theorem using the whiteboard. When the students are working in pairs and interact with each other as well as with the Geogebra software, the teacher facilitates the interactions.

Activities

1.1: Diagnostic assessment- Recall prior knowledge

Phase 2: Start with a challenging problem/question

Description of phase: In the first activity the teacher interacts with the whole class while presenting the problematic situation and then, by facilitating the discussion.

In the second activity (where students use the Geogebra software), the students are working in pairs in front of the laptops and the teacher supervises and facilitates the interactions between the students and the interactions of the students with the digital resource of Geogebra.

Activities

2.1: Help to create secure goal posts in your school (presentation of the problem)

2.2: Help to create secure goal posts in your school (mathematical abstraction/modelling)

Phase 3: Write your hypotheses and present them to the class

Activities

3.1: Formulate hypotheses about the underlying rules

3.2: Present your work to the whole class

Phase 4: Revisiting the challenge

Description of phase: The students revisit the initial challenge and apply the newly-acquired knowledge in order to solve this real-life problematic situation. They write their answers and present them to the whole class. The various answers are compared and contrasted and through a socratic dialog the teacher guides the students to effectively tackle with the problem.

Also, if some students have attempted to answer the question in the first place during phase 2 (maybe intuitively or because they are gifted students or because they already knew the theorems through extracurricular activities or through self-study) they are asked by the teacher reflect upon their initial answers by comparing and contrasting their initial answers with their final answers.

Activities

4.1: So, can you help with the problem about goal posts?

Phase 5: Apply your knowledge

Description of phase: In this phase, the students try to apply what they have learnt so far. They work in pairs and share their answers with the whole class.

Activities

5.1: Apply your knowledge to solve this exercise

Phase 6: Application of new knowledge in a different context

Description of phase: Students read the problem statement and view the accompanying image in the slide. They use their abstract thinking in order to apply what they have learnt and compose a solution to the problem with the ladder (i.e. to find the length of the metal segment ΛΓ that is being used to support the ladder).

Then they read another exercise where they have to apply the reverse theorem and compose a solution.

In both cases, the students report their answers to the teacher. In case of wrong answer, the teacher provides scaffolds or initiates a structured debate where the various answers are compared and contrasted. Finally, one pair of students in each sub-activity demonstrates its solution to the whole-class.

Activities

6.1: Application of new knowledge in a different context--Transfer of knowledge in a similar problem

Phase 7: Address the needs of the gifted students

Description of phase: This is an extra phase for the gifted students, suitable for mixed ability classrooms.

Activities

7.1: An extra activity to address the needs of the gifted students

Phase 8: Create your own problems

Description of phase: Students work in pairs and synthesize their knowledge in order to create a problem that could be solved with the application of the triangle theorem (or the reverse theorem). For this purpose they can use a software or Geogebra software and a word processor software.

Then they have to justify why and how the application of the newly-acquired knowledge solves their problem. Their problem has to be original.

They can search the internet only to find images. Alternatively, they could search images from online databases or form the image galleries built-in various computer applications (Excel, Kidspiration and others)

Activities

8.1: Create your own problems

Phase 9: Self-study

Activities

9.1: Self-study/individual study

Language

English

Grade & Age

Age: 14-15 years old
Grade: secondary education

Domain

Mathematics>Geometry

Keywords/subject

Triangle theorem, dynamic geometry, median of triangle, real life problems

Prerequisites

Prerequisites in terms of ICT-tools:
Students need to be familiarized with the functionalities of the Geogebra viewer
Domain-specific prerequisites:
• Students need to recognize when a triangle is isosceles, especially in conjunction with its functional elements (dichotomos, median, height).
• Student needs to understand that if a line is perpendicular to a line (ε1), then it will also be perpendicular to any other line which is parallel to line ε1.
• Students need to recall the Thales theorem

Difficulty

medium

Learning environment

Computer-based, Lecture-based, Simulator

Duration

1 Hours & 30 Minutes

Teaching approach

Cognitivist: Direct instruction, Collaborative learning, Problem based
Constructivist: Socratic instruction, Experiential learning

Assessment strategy

Diagnostic-assessment, Peer-assessment, Summative assessment

Phases

1.Diagnostic phase

Description of phase

There is a laptop above every desk and the students are working mostly in pairs, except for the proof of the theorem, where a student with the guidance and support of the teacher demonstrates to the whole class the proof of the theorem using the whiteboard. When the students are working in pairs and interact with each other as well as with the Geogebra software, the teacher facilitates the interactions.

Learning activities

1.1 Diagnostic assessment- Recall prior knowledge

Description: The learning objectives of this activity are:

1. To identify when a triangle is isosceles, especially in conjunction with its functional elements (dichotomos, median, height).

2. To understand that if a line is perpendicular to a line (ϵ_1), then it will also be perpendicular to any other line which is parallel to line ϵ_1.

3. To remember the Thales theorem, in specific its application in a triagle

For the first two learning objectives: the students view the powerpoint slides and through a questions-and-answers technique the objective is achieved. If needed, the students also take notes.

For the third learning objective: the students manipulate the functionality of the Geobebra viewer in order to experiment: make a hypothesis, model through a mathematical type and articulate reasoning.

Tools

Software Tools: Presentation software Computer applications (http://www.geogebra.org/)

Hardware Tools: Computer, Projector

Resources

Educational objects (as file):

1. Geogebra file

2. powerpoint file

Duration: 10 Minutes

Phases

2.Start with a challenging problem/question

Description of phase

In the first activity the teacher interacts with the whole class while presenting the problematic situation and then, by facilitating the discussion. In the second activity (where students use the Geogebra software), the students are working in pairs in front of the laptops and the teacher supervises and facilitates the interactions between the students and the interactions of the students with the digital resource of Geogebra.

Learning activities

2.1 Help to create secure goal posts in your school (presentation of the problem)

Description: The teacher presents to the whole class the following problematic situation: "the goal posts at the soccer playground of the school are not secure. We need to make them more secure, by putting four extra metal segments (ΒΓ, ΑΜ and Β′Γ′, Α′Μ′). Firstly, we need to calculate their length. Can you help?".

Tools

Software Tools: Presentation software

Hardware Tools: Computer, Projector

Resources

Educational objects (as file):

1. powerpoint file

Duration: 5 Minutes

2.2 Help to create secure goal posts in your school (mathematical abstraction/modelling)

Description: , the students interact with each other (in pairs) and with a digital resource using the Geogebra software. The problem has three phases/questions:

• Step 1: students experiment with the values of λ which defines the ratio MB/MΓ. They need to discover for which value of λ the segments AM, MB, MΓ are equal.

Then, they are called to formulate a rule about the relationship of the median of the triangle in conjunction with the hypotenuse.

Step 2: For λ=0.5, the students observe the segment MΔ which is perpendicular to the side AΓ. Since M is in the middle of BΓ and MΔ//BA, then MΔ is the median and the height of the triangle AMΓ. This means that the triangle AMΓ is isosceles.

• Step 3: the students experiment with the value of the degrees of angle A in order to investigate whether the above rule is true for another angle different than the right angle. Thus, the guided discovery approach is used in order to help the student reverse the above theorem (see step 1).

Tools

Software Tools: Computer applications (http://www.geogebra.org)

Resources

Educational objects (as file):

1. geogebra file

Phases

3.Write your hypotheses and present them to the class

Learning activities

3.1 Formulate hypotheses about the underlying rules

Description: In this learning activity the students, based on what they learnt so far, will formulate hypotheses about the underlying rules (i.e the theorem and the reverse theorem). Sentence openers are used to guide them. They are working in pairs. They can use a digital resource created in Geogebra to help them visualize the rules.Resources

Educational objects (as file):

1. powerpoint file

2. Geogebra file

Duration: 7 Minutes

3.2 Present your work to the whole class

Description: Students present their work to the whole class and if a a pair fails to formulate the right hypothesis a structured debate is initiated from comparing and contrasting the students' answers.

Duration: 3 Minutes

Phases

4.Revisiting the challenge

Description of phase

The students revisit the initial challenge and apply the newly-acquired knowledge in order to solve this real-life problematic situation. They write their answers and present them to the whole class. The various answers are compared and contrasted and through a socratic dialog the teacher guides the students to effectively tackle with the problem.
Also, if some students have attempted to answer the question in the first place during phase 2 (maybe intuitively or because they are gifted students or because they already knew the theorems through extracurricular activities or through self-study) they are asked by the teacher reflect upon their initial answers by comparing and contrasting their initial answers with their final answers.

Learning activities

4.1 So, can you help with the problem about goal posts?
Description: sentence openers
Tools
Software Tools: Presentation software
Hardware Tools: Computer, Recorders

Resources

Educational objects (as file):
1. powerpoint file

Duration: 5 Minutes

Phases

5.Apply your knowledge

Description of phase

In this phase, the students try to apply what they have learnt so far. They work in pairs and share their answers with the whole class.

Learning activities

5.1 Apply your knowledge to solve this exercise
Description: The main learning objective of this activity is to apply the triangle theorem in order to solve this exercise. Thus, students read the exercise (by selecting the first cube) and try to apply what they have learnt so far in order to solve the exercise. They work in pairs and share their answers with the whole class.
Tools
Software Tools: Web browsers
Hardware Tools: Computer

Resources

Educational objects (as file):
1. web page

Duration: 10 Minutes

Phases

6.Application of new knowledge in a different context

Description of phase

Students read the problem statement and view the accompanying image in the slide. They use their abstract thinking in order to apply what they have learnt and compose a solution to the problem with the ladder (i.e. to find the length of the metal segment ΛΓ that is being used to support the ladder).
Then they read another exercise where they have to apply the reverse theorem and compose a solution.
In both cases, the students report their answers to the teacher. In case of wrong answer, the teacher provides scaffolds or initiates a structured debate where the various answers are compared and contrasted. Finally, one pair of students in each sub-activity demonstrates its solution to the whole-class.

Learning activities

6.1 Application of new knowledge in a different context--Transfer of knowledge in a similar problem
Description: The learning objectives of this activity are:
•⬜To apply the newly-created knowledge in a different context
•⬜To transfer the newly-acquired knowledge in similar problems
Resources

Educational objects (as file):
1. powerpoint file

Duration: 10 Minutes

Phases

7.Address the needs of the gifted students

Description of phase

This is an extra phase for the gifted students, suitable for mixed ability classrooms.

Learning activities

7.1 An extra activity to address the needs of the gifted students
Description: Students read the problem statement and view the slide where it is presented and then they try to compose a solution to the

problem. Students work individually or in pairs.The teacher plays the role of the facilitator.
Tools
Software Tools: Presentation software
Hardware Tools: Computer

Resources

Educational objects (as file):
1. powerpoint file

Duration: 10 Minutes

Phases

8.Create your own problems

Description of phase

Students work in pairs and synthesize their knowledge in order to create a problem that could be solved with the application of the triangle theorem (or the reverse theorem). For this purpose they can use a software or Geogebra software and a word processor software. Then they have to justify why and how the application of the newly-acquired knowledge solves their problem. Their problem has to be original. They can search the internet only to find images. Alternatively, they could search images from online databases or form the image galleries built-in various computer applications (Excel, Kidspiration and others)

Learning activities

8.1 Create your own problems
Description: The resources will be created by the students. The role of the teacher is to facilitate and supervise the collaborative learning process.
Tools
Software Tools: Word processor Presentation software Search engines Web browsers Online databases Computer applications (Geogebra dynamic geometry appplication, http://www.geogebra.org), Text, image, audio or video viewer
Hardware Tools: Computer

Duration: 20 Minutes

Phases

9.Self-study

Learning activities

9.1 Self-study/individual study
Description: Students gather information for self-study material (at their homes), it involves today's lesson material and more exercises on the topic. With this material the teacher aims to assess the learning of each student individually. The resources are available online at:

https://sites.google.com/site/gloriayiasemis

https://www.engrade.com/teacher/?rad=929855

Tools

Software Tools: Word processor Presentation software Web browsers Computer applications (http://www.geogebra.org)

Hardware Tools: Computer

Resources

Educational objects (as file):

1. presentation file

2. geogebra file

Educational objects (as url):

1. Exercises for self-study can also be downloaded from this website